Académie d'Orléans-Tours

Baccalauréat Général – Session 2010

Épreuve spécifique des sections européennes

Anglais / Mathématiques

Les candidats restituent les textes à l'issue de leur épreuve.

SUJET 3

Arithmetic

I) Introduction

From little problems like " A workman has a 330 cm by 264 cm piece of fabric. He'd like to cut it into identical squares, avoiding waste" to modern ciphers using public key such as the RSA, you cannot do without prime numbers, in general, and the unique-prime-factorization theorem, also known as the fundamental theorem of arithmetic, in particular.

The earliest surviving records of the explicit study of prime numbers come from the Ancient Greeks. Euclid's Elements contain important theorems about primes, including the infinitude of primes and the fundamental theorem of arithmetic. The sieve of Eratosthenes, attributed to Eratosthenes, also dates back this period.

After the Greeks, little happened with the study of prime numbers until the 17th century when Pierre de Fermat stated Fermat's little theorem, later proved by Leibniz and Euler.

II) Questions

1) Explain what a prime number is.

2) In this question, we'll consider the little problem.
   a) Give all the common factors of 330 and 264.
   b) What are the possible sizes of the squares ?
   Deduce the size of the largest square.
   What does this number represent to 330 and 264 ?

3) Explain two different ways to simplify the following fraction : $\frac{21450}{60775}$.

SUJET 4

History of secret codes

I) Introduction

The science of cryptography has often changed the course of history, from the well-known Caesar cipher to modern communication encryption algorithm, with the never-ending development of code-making and code-breaking.

From the 9th century, ciphertexts produced by substitution ciphers or any monoalphabetic cipher could too often be broken using frequency analysis perhaps discovered by Al-Kindi. The Vigenère cipher is one of the truly great breakthroughs in the development of cryptography, in the second half of the 16th century. This extension of Alberti's work on polyalphabetic ciphers brought such a strong cipher that it is sometimes called "the unbreakable cipher", even though Babbage, using extended frequency analysis techniques, in the mid 1800s, found a way to break it.

However, these types of ciphers were still in use on the German side during the second world war, with the Enigma machine.

II) Questions

1) Explain what encryption techniques Caesar used. You could encrypt for example "CAESAR SHIFT CIPHER", using the original three places shift.

2) Explain how basic frequency analysis can help breaking monoalphabetic ciphers.

3) Do you know a substitution code which would resist frequency analysis ?

4) Explain why ciphers are so important in our everyday life.

SUJET 6

Pie charts

A scientific revolution began at the end of the 18th century with the creation and popularization of the graphic display of data by Scottish inventor William Playfair, who introduced the line graph, bar chart, and pie chart into statistics.. His remarkable Atlas demonstrated how much could be learned if one plotted data graphically and looked for suggestive patterns to provide evidence for pursuing research. In the Statistical Breviary, Playfair invented the pie chart and expanded upon this concept to falicitate the comparison of the resources of European countries. In a pie chart, the arc length of each sector and consequently its central angle and area is proportional to the quantity it represents.

(absoluteastronomy.com)

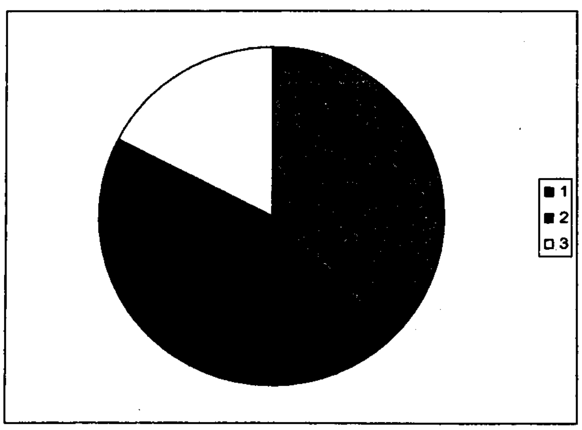
Exercise :

A survey is organised among a sample constituted of 1040 people.

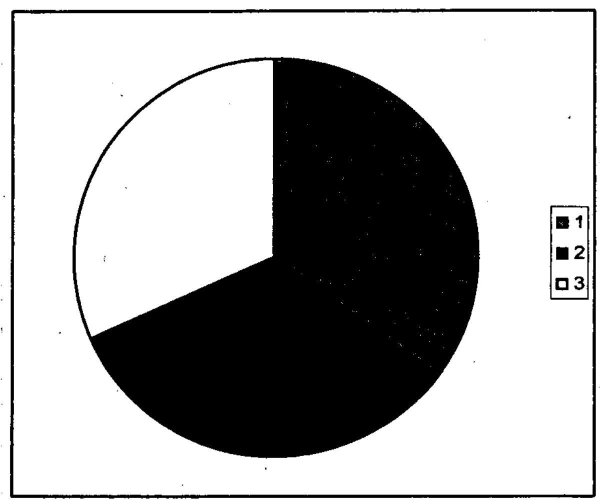
The sample is splitted into two groups : the group A is composed by 320 women, where as the group B is only composed by men.

The following pie charts represent the voting intention before a general election.

Group A



Group B



	Labour	Conservative	Other parties
Group A	135°	162°	
Group B		120°	114°
Total			

(a) Calculate the number of vote intentions for the other parties in Group A.
(b) Calculate the total number of vote intentions collected in Group B.
(c) The journalist wants to gather the voting intentions into one Group.
   a. Give the voting intentions for the Labour (give a percentage) in the sample
   b. Is this sample representative of the electors ? Justify you answer.

SUJET 7

Gravity

Sir Isaac Newton (1643 –1727) was an English physicist, mathematician, astronomer, natural philosopher, alchemist, and theologian who is considered by many scholars and members of the general public to be one of the most influential men in human history. His 1687 publication of the *Principia* is considered to be among the most influential books in the history of science, laying the groundwork for most of classical mechanics. In this work, Newton described universal gravitation and the three laws of motion which dominated the scientific view of the physical universe for the next three centuries. Newton showed that the motions of objects on Earth and of celestial bodies are governed by the same set of natural laws by demonstrating the consistency between Kepler's laws of planetary motion and his theory of gravitation, thus removing the last doubts about heliocentrism and advancing the scientific revolution.

(Wikipedia)

Note: The gravity constant on the earth g is approximately

Exercise:

The height, h metres, of a sky rocket t seconds after being launched is given by the formula

$h(t) = at^2 + bt + 2$

where a and b are constants. The heights of the rocket above the ground at two different times are given in the table below

t (seconds)	1	2
H (metres)	37	62

(a) At what height above the ground is the rocket launched?

(b) (i) Use the table of values to show

$$a + b = 35$$

and $$4a + 2b = 60$$

(ii) Solve these simultaneous equations to find the value of a and the value of b.

(c) Usually, the equation is written in the general form where g is the gravity constant, v_0 is the vertical speed of the rocket at t=0, $h_0 = h(0)$ its initial height. Can you link this formula with your answer to the question (b) (ii)?

SUJET 12 **Thème : Suites**

Calculatrice autorisée

PART N°1

All population growth, from bacterial division to human procreation are examples of exponential growth.
Nearly all populations will tend to grow exponentially as long as there are available resources.
Most populations have the potential to expand at an exponential rate, since reproduction is generally a multiplicative process.
Any species growing exponentially under unlimited resource conditions can reach enormous population densities in a short time. Darwin showed how even a slow growing animal like the elephant could reach an enormous population if there were unlimited resources for its growth in its habitat.
But humans have changed ecosystems as they searched for food, fuel, shelter and living space.
Human destruction of habitats and species through direct harvesting, pollution, atmospheric changes, and other factors is threatening current global stability, and, if not addressed, ecosystems will be irreversibly affected.

Adapted from various internet sources

PART N°2

The frog population in a pond is decreasing dangerously. Therefore some ecologists are worrying and try to count this population on each first monday of november.

Year	2005	2006	2007	2008
Year rank	0	1	2	3
Frog population	1000	900	810	729

People assume that the decrease is following a geometric sequence.
Let V_n be the population in year rank n, with V_0=1000.

1. Compute the common ratio.
2. Express v_n in term of n.
3. What will the population be in 2020 if the decay is follows the same model?
4. In 2010, after some works around the pond, the frog population increases at the rate of 41 frogs each year. Assuming this new sequence is labelled u_m where m is the rank of the year after 2010. (m=0 in 2010)
   a) What sort of sequence is this ?
   b) Express u_m in term of m. (remember that V_0=V_5
   c) In what year (2010 + m), will the frog population reach the same level as in 2005?

SUJET 13
Thème : Géométrie de Collège

PART N°1

Every time you graph an equation on a Cartesian coordinate system, you are using the work of René Descartes. Descartes, a French mathematician and philosopher, was born in La Haye, France in 1596. Descartes was an outstanding student at La Flèche, especially in mathematics.

The philosophical system that Descartes developed, known as Cartesian philosophy, was based on skepticism and asserted that all reliable knowledge must be built up by the use of reason through logical analysis.

Descartes published various treaties about philosophy and mathematics. In 1637, Descartes published his masterwork, « Discourse on the Method ». In a part called « La Géometrie », Descartes described what is now known as the system of Cartesian Coordinates, or coordinate geometry. In Descartes's system of coordinates, geometry and algebra were united for the first time to create what is known as analytic geometry.

Adapted from http://www.bookrags.com/research/descartes-and-his-coordinate-system-mmat-02/

PART N°2

The grid below represents the radar screen of the control tower. The origin is the control tower. Each centimetre represents one kilometre.

1. Plot the waypoints : M (- 5 ; 1) L (- 2 ; 4) and K (3 ; 2).

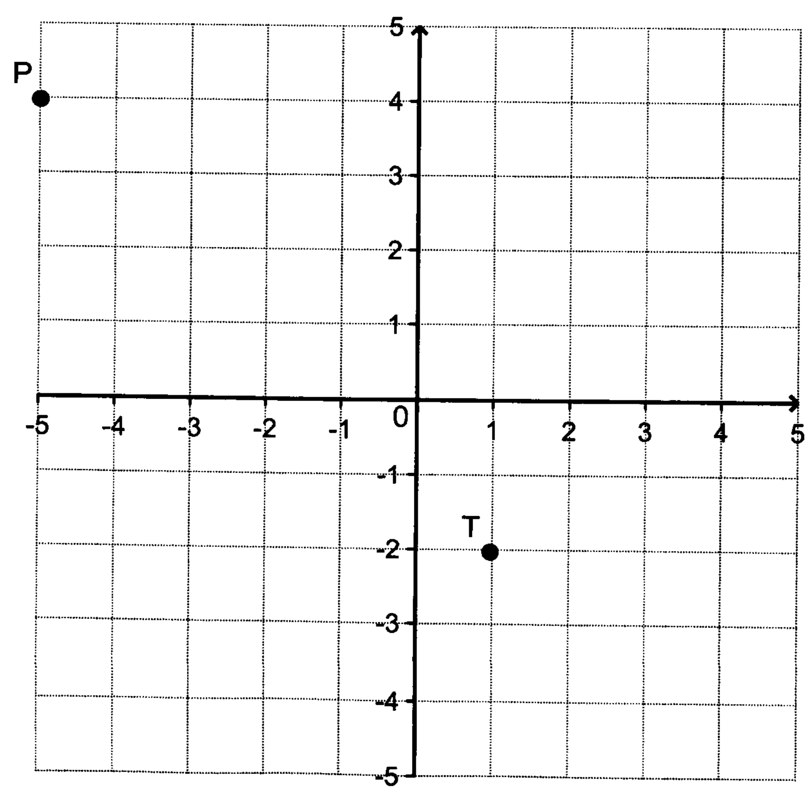
2. A plane appeared in P. The air traffic controller asks the captain to fly to point T.
   a) Draw in green the path the plane has to follow.

   b) Compute the length of segment [PT].

   c) What does the line (PT) represent for the segment [LM] ?

3. Now the plane has to turn and to follow a path (d) which is parallel to (KO). Draw (d). What is the equation of this line (d) ?

4. The landing occurs at the intersection A between this path and the x-axis. Plot this point A. Compute its coordinates.



SUJET 20

PROBABILITY

I. What is a model?

Many people travel to work each day by train. You may have played with a toy train set when you were younger. Your toy train was a model of a real train. It was like the real train in many ways but did not have *all* the features of the real thing.

In the same way that a model train can help to solve problems in the real world so a mathematical model can be used to find solutions to problems without the need to construct a physical model.

Starting from a real-world problem, a mathematical model is devised and this is used to make predictions about the expected behavior of the real-world problem.

from STATISTICS 1, by Greg Attwood and Gordon Skipworth

II. Exercise

A bag contains five coins: one 50p, one 10p, one 20p, one 2p and one 5p. You place your hand in the bag and pick out the first coin you feel.

1. Let's consider that the issues are equally likely. Give the probability for each issue

2. An experiment is carried out on the same basis. The results are given in the following table:

Coin	2p	5p	10p	20p	50p
Relative frequency (in %)	23.8	10.9	21.8	16.7	26.8

What do you notice? Do you think we can choose the equally likely model?

A new table is given with mass and diameter of each coin

coin	2p	5p	10p	20p	50p	Total
diameter (mm)	25.9	18	24.5	21.4	27.3	
mass (g)	7.12	3.25	6.5	5	8	

Find a new relation to compute the probabilities given above.

SUJET 21

Tides: from observation to prediction

Tides are the cyclic rising and falling of the Earth's ocean surface caused by the tidal forces of the Moon and Sun acting on the oceans. Isaac Newton laid the foundations for the mathematical explanation of tides in the *Philosophiae Naturalis Principia Mathematica* (1687). In 1740, the Académie Royale des Sciences in Paris offered a prize for the best theoretical essay on tides. Daniel Bernoulli, Antoine Cavalleri, Leonhard Euler, and Colin Maclaurin shared the prize. The first major theoretical formulation for water tides was made by Pierre-Simon Laplace, who formulated a system of partial differential equations relating the horizontal flow to the surface height of the ocean. The Laplace tidal equations are still in use today.

(From *New World Encyclopedia*, Paragon House Publishers)

The Rule of Twelfths is a rule of thumb for approximating a harmonic phenomenon (such as the height of the tide as a function of time). It works as follows:

x	0	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$
$f(x)$	0	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{6}{12}$	$\frac{9}{12}$	$\frac{11}{12}$	$\frac{12}{12}$

1. Draw a dot plot of these data on a coordinate plane (a scale of 12 cm for one unit is recommended). Join the dots by a polygonal line in order to represent a piecewise linear function.

2. Using trapezoids (or rectangles and right angled triangles), compute the area enclosed between this polygonal curve, the x-axis , $x = 0$ and $x = 0{,}5$ lines .

3. It has been proved that the function g defined by $g(x) = A[1-\cos(2\pi x)]$ is closely approximated by the previous polygonal curve. Find the value of A so that g and f coincide in $x = 0{,}5$

4. Compute $\int_0^{0,5} g(x)dx$ and compare with the result obtained in question 2.

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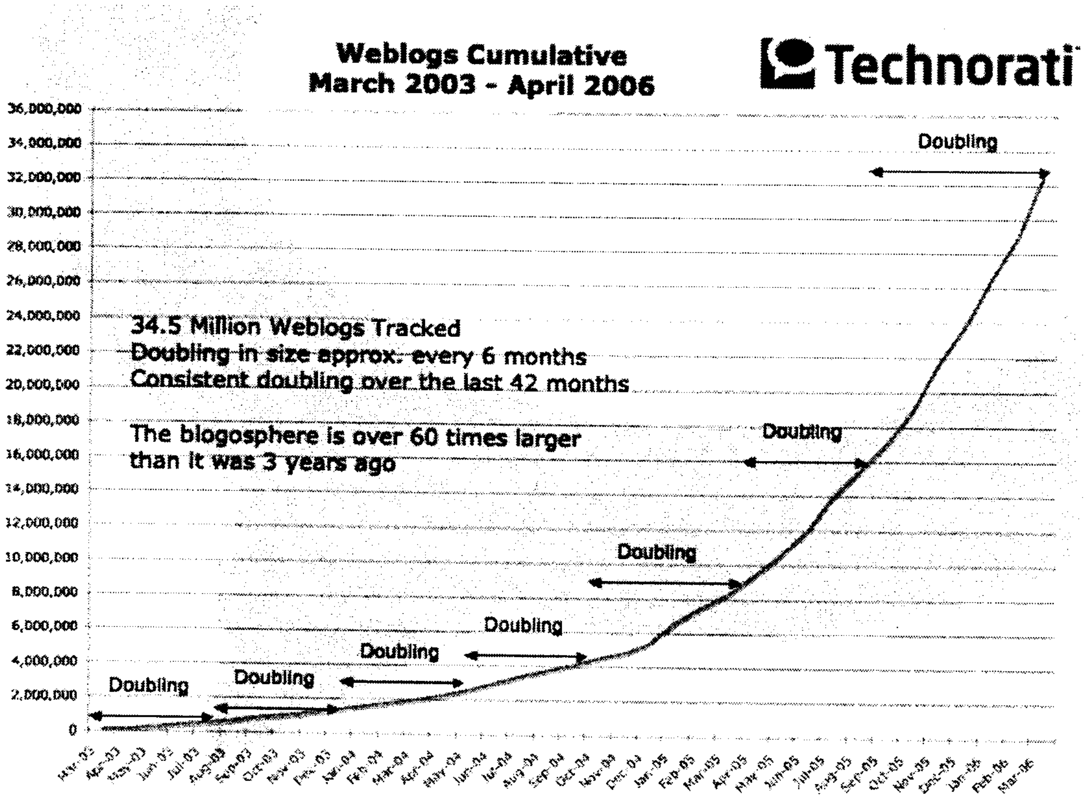
SUJET 22

Thème : Fonctions exponentielles
CALCULATRICE AUTORISÉE

I Première partie

The state of the blogosphere

A Blog (a contraction of the term "Web log") is a Web site, usually maintained by an individual with regular entries of commentary, descriptions of events, or other material such as graphics or video. The Blogosphere is the collective community of all blogs. Discussions "in the Blogosphere" have been used by the media as a gauge of public opinion on various issues.
According to Technorati (an Internet search engine for searching blogs), the number of blogs doubled about every 6 months between march 2003 and march 2006, as shown on the chart below:



This is an example of the early stages of logistic growth, where growth is approximately exponential, since blogs were a recent innovation at that time. However, as the number of blogs approaches the number of possible producers (humans), saturation should occur, growth decline, and the number of blogs eventually stabilize.

from Technorati web site and Wikipedia

SUJET 22 (suite)

II Deuxième partie

1°) Given that there were 271 000 blogs in april 2003 calculate how many there were six months later, and then in april 2006 (i.e. 42 months later). Give the result to the thousand.

2°) a) Find an equality in terms of x (t + 6) and x(t).

b) if x represents the number of blogs at a given time t, x depends exponentially on time t. We assume that :

$x(t) = ab^{\frac{1}{6}}$ with t expressed in months.

Calculate the constant b (known as the "growth factor")

c) If we chose to set t=0 in april 2003, what should be the value of the constant a?

3°) We now have fully determined the function. Calculate the number of blogs :

a) in april 2008

b) in june 2010

c) what comments would you make ?

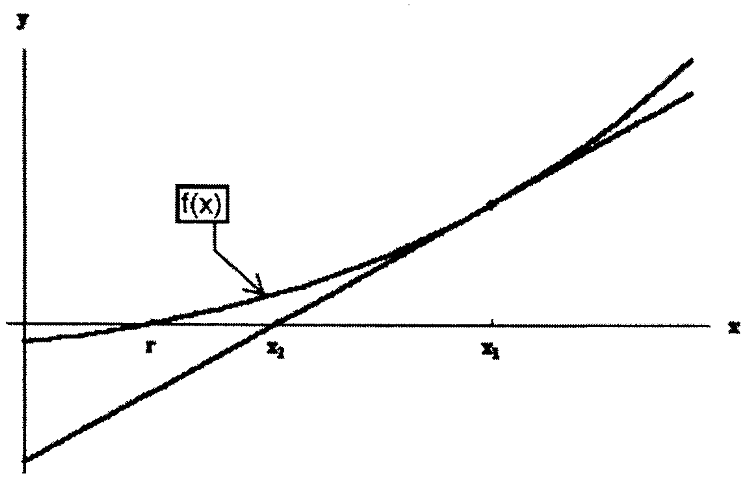
SUJET 24

3Thème : dérivées [calculatrice nécessaire]

I-The Newton – Raphson Method

Newton and Raphson used ideas of the Calculus to generalize a method to find the zeros of an arbitrary equation : $f(x)=0$ where f is an arbitrary function.

Let r be a root (also called a "zero") of $f(x)$, that is $f(r)=0$. Assume that $f'(r)\neq 0$. Let x_1 be a number close to r (which may be obtained by looking at the graph of f). The tangent line to the graph of f at $(x_1, f(x_1))$ has x_2 as its x-intercept.



Easy calculations give $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$. Since we assumed $f'(r)\neq 0$, we will not have problems with the denominator being equal to 0. We continue this process and draw the tangent line to the graph of f at $(x_2, f(x_2))$. It has x_3 as its x-intercept. We find x_3 through the equation $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$. This process will generate a sequence of numbers (x_n) which approximates r.

[adapted from varioux sources]

II-An example

Let f be the function $f(x)=x^3-4x+1$ and let us try to approximate a root of $f(x)=0$.

1. Usin a scientific calculator, obtain the graph of f. Where are the roots of f located ?
2. Calculate the derivative of f. Does $f'(x)$ vanish between 0 and 1 ?
3. Assuming $x_1=0$, compute x_2, x_3 and x_4. Check if x_4 is a good approximation of a solution.