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ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 3

Theme : inequalities, probability

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Are "proofs without words" really proofs? Some mathematicians consider such visual arguments to be of little value, and that there is one and only one way to communicate mathematics, and "proofs without words" are not acceptable. But to counter this viewpoint, some other mathematicians state that to be a scholar[1] of mathematics, you must be born with the ability to visualize. Most teachers try to develop this ability in their students. "Draw a figure..." is classic teaching advice. Einstein and Poincaré's[2] views that we should use our visual intuition, are well known.

So, if "proofs without words" are not proofs, what are they? This question does not have a simple, concise answer. But generally, these "proofs" are pictures or diagrams that help the observer see why a particular statement may be true, and also to see how we can prove it is true.

Adapted from "Proofs without words" by Roger B. Nelsen.(The Mathematical Association of America, 1993)

1. Read the first three lines to " …not acceptable."
2. Explain what the text deals with and comment on it.

Exercise.

A box and an urn contain red and blue balls. There are $b=10$ balls in the box and $a=3$ of them are red. There are $d=13$ balls in the urn and $c=4$ of them are red.

1. Draw a picture in order to visualize the data.
2. We pick a ball out of the urn or of the box at random.
   - **a.** What is the probability of getting a red ball given that the ball is picked out of the box?
   - **b.** What is the probability of getting a red ball given that the ball is picked out of the urn?
3. Now all the $b+d=23$ balls are gathered in the box. We pick a ball out of the box at random.
   - **a.** What is the probability of getting a red ball?
   - **b.** Compare this result to the probabilities from question 2. Is it surprising?

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[1] A scholar: *a person who studies a subject in great detail, especially at a university.*
[2] Poincaré: *Henri Poincaré was a French mathematician, theoretical physicist, engineer, and a philosopher of science. (1854-1912)*

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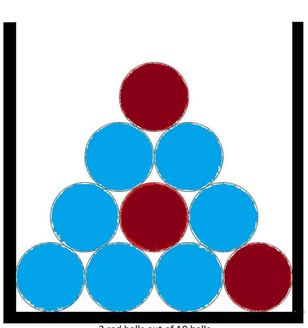
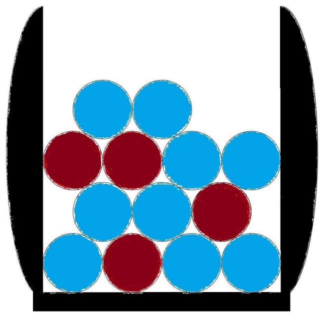
SUJET 3 - CORRIGÉ

3.



3 red balls out of 10 balls



4 red balls out of 13 balls

a.

b.

i. $\frac{3}{10}$

ii. $\frac{4}{13}$

c.

i. $\frac{7}{23}$

ii. $\frac{3}{10} < \frac{7}{23} < \frac{4}{13}$. La dernière probabilité est comprise entre les 2 précédentes. Ce résultat était prévisible car la proportion de boules rouges dans le mélange doit logiquement être intermédiaire entre les 2 précédentes proportions.

Éléments à prendre en compte pour évaluer la capacité d'analyse et d'argumentation :

- Il existe 2 points de vue différents parmi les mathématiciens à propos des "preuves sans mot"
- Ces "preuves" n'en sont pas vraiment, mais elles sont tout de même très utiles.

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SUJET 4

Phi an irrationnal number
Thème : Nombres

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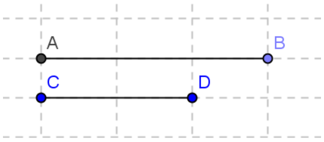
Asking whether square roots are fractions is linked to the theory of measurement as known to the ancient Greeks. Suppose we have a line AB whose length we wish to measure, and an indivisible 'unit' CD with which to measure it. To make the measurement, we place the unit CD sequentially against AB. If we place the unit down m times and the end of the last unit fits exactly with the end of AB (at the point B), the length of AB will simply be m. If not, we can place a copy of AB next to the original and carry on measuring with the unit. The Greeks believed that, at some point, using n copies of AB and m units, the unit would fit exactly with the end-point of the m^{th} AB. The length of AB would then be m/n.

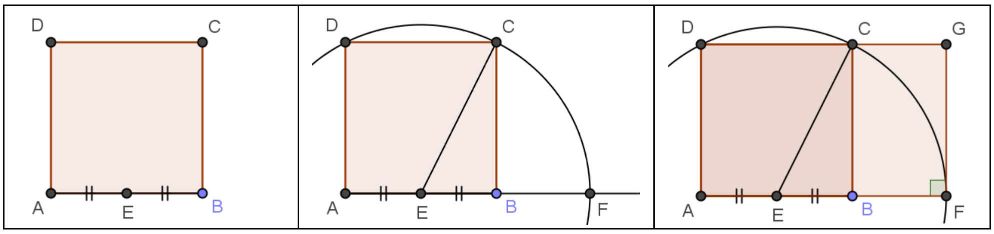


Extract from "50 mathematical ideas" by Tony Crilly

1. Read the first three lines of the text ending with 'to measure it'.
2. Explain what the text deals with and comment on it.

Exercise

1. ABCD is a square. Explain how E, F and G are built.



2. Assume that AD = 1 and let Φ be the number $\Phi = \dfrac{1+\sqrt{5}}{2}$ (the golden number).

   A golden rectangle is a rectangle so that $\dfrac{length}{width} = \Phi$.

   a. Work out EC, and then AF.

   b. Explain why AFGD is a golden rectangle.

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MATHÉMATIQUES – ANGLAIS

SUJET 4- CORRIGÉ

Phi an irrationnal number
Thème : Nombres

Éléments à prendre en compte pour évaluer la capacité d'analyse et d'argumentation :

- Definition of what an irrational number is.
- Explain how the figure illustrates the text.
- Some knowledge about Mathematics in ancient Greece is expected.

Exercise

1 – E is the midpoint of AB.

F is the intersection point of the circle of centre E and radius EC with the half-line AB.
G is the intersection point of the perpendicular to AB through F with line DC.

2.a ABC is a right-angled triangle at B. According to the Pythagorean theorem :

$EC^2 = BC^2 + EB^2$

$EC^2 = \frac{5}{4}$. So, $EC = \frac{\sqrt{5}}{2}$.

$EC = EF$, because they are both radii of the same circle.

So, $AF = AE + EF = \frac{1+\sqrt{5}}{2}$

2.b $\frac{AF}{AD} = \frac{\Phi}{1} = \Phi$. AFGD is then a golden rectangle.

3 (extension possible) There is another golden rectangle in that configuration. Which one?

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MATHÉMATIQUES – ANGLAIS

SUJET 5

Largest known Prime Number Discovered in 2013
Thème : Arithmétique

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

On January 25th 2013, a team of mathematicians led by Dr Cooper (University of Central Missouri) have discovered the largest known prime number yet : $(2^{57,885,161} - 1)$, a number so large that it has over 17,400,000 digits.

Why does it matter? That's an immensely huge number, but *why should anyone care*?

Prime numbers are absolutely central to mathematics, they are the elementary particles of the mathematical world. Just as matter is made up of protons, electrons and other particles, naturals[1] are made up of primes.

Even if you don't care about mathematics, you still use primes without knowing it. How? With encryption. Every time you buy something securely off the internet or send a credit card number, you depend on primes.

For mathematicians, there is also a certain amount of glory involved. The greatest mathematical minds of all time have all done work on prime numbers, from Euclid to Leonard Euler, to Cooper. It's part and parcel of mathematics.

Adapted from : David Self Newlin – Science and Innovation – KSL.com

1. Read the first paragraph to «digits».
2. Explain what the text deals with and comment on it.

Exercise

1. What is a prime number?
2. TRUE or FALSE?

   For each of the following statements, determine whether it is true or false.

   For true statements, justify briefly your opinion.

   For false statements, find out a counterexample.

   a. Odd numbers are all prime.

   b. Only one even number is prime.

   c. For any natural number n, $(6\,n - 1)$ is prime.

   d. 91 is a prime number.

   e. For any number x, there is a prime number greater than x.

[1] A natural number is a whole number which is greater than or equal to 1.

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SUJET 5 - CORRIGÉ

Largest known Prime Number Discovered in 2013
Thème : Arithmétique

3. A prime number is a natural number which has exactly 2 factors (1 and itself).

4. TRUE or FALSE?

a. *Odd numbers are all prime.* **FALSE**	Contre-exemples évidents...
b. *Only one even number is prime.* **TRUE**	On pourra demander une justification... éventuellement guider le candidat : *What is an even number ?* 2 is the only prime which is even.
c. *For any natural* number n, is prime.* **FALSE**	On peut remarquer que si c'était le cas, il n'y aurait pas grand mérite à trouver de nouveaux nombres premiers ! $6\times6-1=35$ and 35 is not prime.
d. 91 is a prime number. **FALSE**	Si nécessaire, encourager le candidat à tenter de diviser 91 par 2, par 3, … $91=7\times13$ therefore 91 is not prime.
e. *For any number x, there is a prime number greater than x.* **TRUE**	Dans le cas où le candidat ignore ce résultat, on pourra faire remarquer que, d'après le texte, "battre le record" précédent du plus grand nombre premier a été, reste et restera un challenge parmi les mathématiciens... ce qui permet de conjecturer… The set of primes is infinite.

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SUJET 7

Game of life
Thème : Algorithmique

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Despite the name of the game, when John Conway developed the system he called *Life*, he wasn't aiming to simulate life at all. Conway's original aim was entirely mathematical - he was trying to find a so-called universal system, that is, a system capable of carrying out arbitrary computations - a sort of infinitely programmable computer.

Life has stimulated a huge amount of public interest ever since it was first publicised in Gardner's column. The US military at one point estimated that millions of dollars worth of computing time had been "wasted" looking at the *Life* game, and it still continues to be played today. Conway is pleased that *Life* interests the public. "I'm always trying to sell mathematics to the general public, to turn them on" he says.

Extract from the webzine "Plus", April 2002 issue (see http://plus.maths.org)

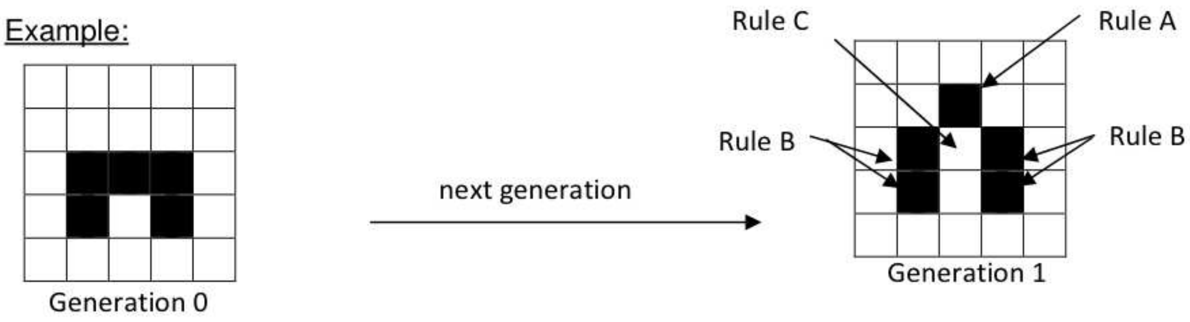
1. Start the interview by reading the first three lines of the text ending with "happen".

2. Explain what the text deals with and comment on it.

Exercise

Life is played on a grid of square cells. A live cell is blackened whereas a dead cell is identified by leaving the square empty. Each cell in the grid has a neighbourhood consisting of the eight cells in every direction including diagonals. After one generation:

Rule A. A dead cell with exactly three live neighbours becomes a live cell (birth).
Rule B. A live cell with either two or three live neighbours stays alive (survival).
Rule C. In all the other cases, a cell dies or remains dead (overcrowding or loneliness).



1. Using the same rules, verify that this example leads to full mortality after 2 more steps
2. Starting from only 3 live cells, find an initial configuration that globally ensures life forever.
3. Find a pattern of 4 live cells that guarantees individual immortality

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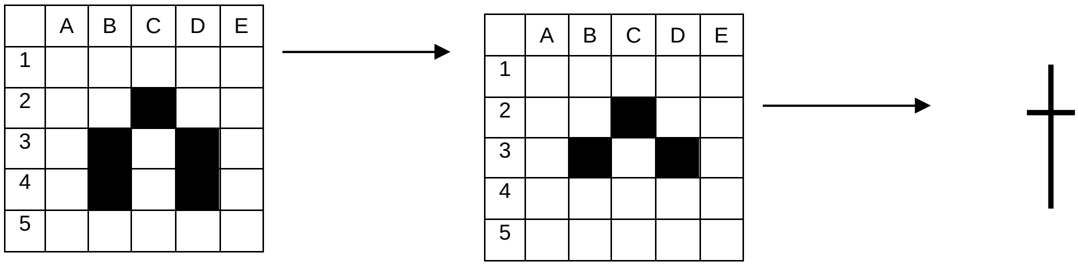
SUJET 7 - CORRIGÉ

Game of life
Thème : Algorithmique

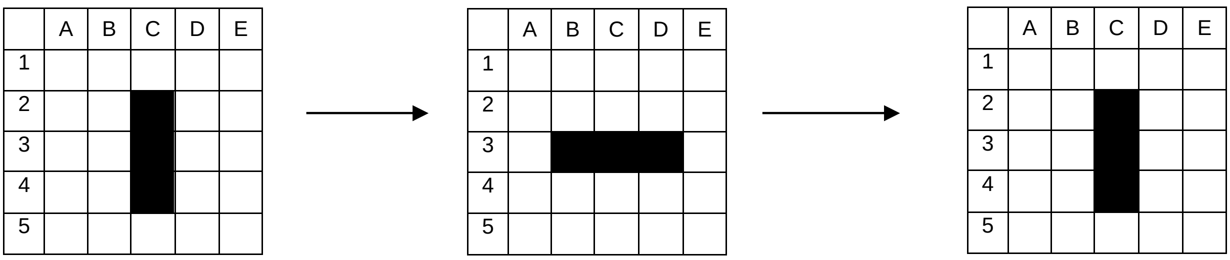
1. Let’s use coordinates like in a chessboard game.

Cells B4 and D4 have only one living cell in their neighbourhood, so they both die. Cells C2, B3 and D3 are surrounded by two living cell, so they survive. There is no birth of new cell. D3 are surrounded by two living cell, so they survive. There is no birth of new cell. At the next generation, all the remaining cells die of loneliness.



Generation 1 Generation 2 Generation 3

2. From last question experience, we have to cluster the initial cells in a “bar-like” pattern. We then observe that we get a periodic “blinking” configuration.



Génération 0 Génération 1 Génération 2

3. If we start with a 2x2 blackened square, we get this same stationary pattern for any incoming generation.
4. Bonus: what’s happening if the initial pattern is like a “plus” symbol?

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SUJET 8

Thème : géométrie

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

What is pi ? First and foremost it is a number, between 3 and 4 (3.14159…). It arises in any computation involving circles : the area of a circle of radius 1 or equivalently, though not obviously, the perimeter of a circle of radius 1/2. The nomenclature π is presumably the Greek letter "p" in periphery. The most basic properties of π were understood in the period of classical Greek mathematics by the time of the death of Archimedes in 212 BC.

The Greek notion of number was quite different from ours, so the Greek numbers were our whole numbers : 1, 2, 3… In Greek geometry the essential idea was not numbers but continuous magnitude, e.g. line segments. It was based on the notion of multiplicity of units and, in this sense, numbers that existed were numbers that could be drawn with just an unmarked ruler and compasses. […]

Unfortunately π is not constructible, though a proof of this would not be available for several thousand years. In this context there isn't a more basic question than "is π a number ?" Of course, our more modern notion of number embraces the Greek notion of constructible and doesn't depend on construction.

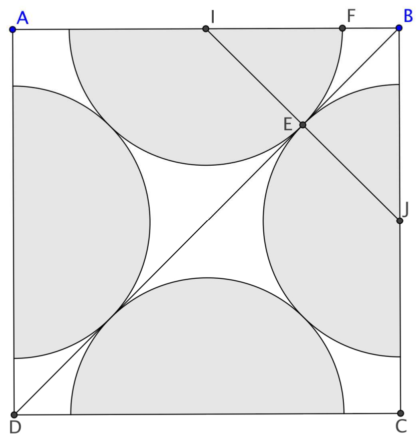
Peter BORWEIN, *The amazing number π*, 2000

1. Read the first four lines of the text down to "of radius ½".
2. Explain what the text deals with and comment on it.

Exercise

On the rough drawing given below, ABCD is a square and the length AB is 2 dm long. I is the middle of the line-segment [AB] and J the middle of [BC]. We assume that both semicircles are tangent at the point E, the middle of the line segment [IJ]. Our goal is to calculate the area of a few shapes on this figure.

1. Calculate the length of the line-segment [AC] using a famous theorem in the right triangle ABC.
2. Calculate the length of the line-segment [IJ] using another classical theorem in the triangle ABC.
3. Prove that the length of [IE] is $\dfrac{\sqrt{2}}{2}$ dm long.
4. Work out the area of one semi disk and then deduce the dark area. What is the part of it compared to the area of the square ?



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SUJET 8 - CORRIGÉ

Thème : géométrie

1. We use Pythagoras theorem in the right triangle ABC, so : $AC^2 = AB^2 + BC^2$, $AC^2 = 2^2 + 2^2 = 4 + 4 = 8$ dm, we can deduce : $AC = \sqrt{8} = 2\sqrt{2}$ dm.

2. We use the middles theorem in the triangle ABC : I is the middle of [AB] and J the middle of [BC], so the length of the line-segment [IJ] is half of AC, thus $IJ = \frac{1}{2}\ 2\sqrt{2} = \sqrt{2}$dm.

3. E is the middle of [IJ] so $IE = \frac{1}{2}\ IJ = \frac{\sqrt{2}}{2}$ dm.

4. The area of one semi disk is : $\frac{1}{2} . R^2 . \pi = \frac{1}{2} . \left(\frac{\sqrt{2}}{2}\right)^2 . \pi = \frac{1}{2} . \frac{1}{2} . \pi = \frac{\pi}{4}$ dm².

   So the brown area is equal to : $4 \times \frac{\pi}{4} = \pi$ dm².

   The area of the square is : $2^2 = 4$ dm²,so the part of the dark area is : $\frac{\pi}{4}$.

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SUJET 9

Box and whisker plot
Thème : **Statistics**

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

As a chemist-turned-topologist-turned statistician, John Wilder Tukey(1915 – 2000) played a key role in the development and study of statistics in the mid 1900's. The field of statistics has benefited tremendously from his contributions. He began a major research movement in graphical methods for data analysis in statistics. Tukey is credited with the invention of many methods, both graphical and numerical, that are extremely effective in statistical applications. Tukey has done work in time series analysis, exploratory data analysis, and multiple comparisons that is considered revolutionary. He has also greatly contributed to the literature available on the philosophy and research of probability and statistics. Three of Tukey's specific contributions are the Box-and-Whisker Plot, the Stem-and-Leaf Diagram, and Tukey's Paired Comparisons. Box-and-Whisker Plots were invented by Tukey as a means to display groups of data. Typically, five values from a set of data are used : the extremes, the upper and lower hinges (quartiles), and the median.

From : Umn stats - morris.

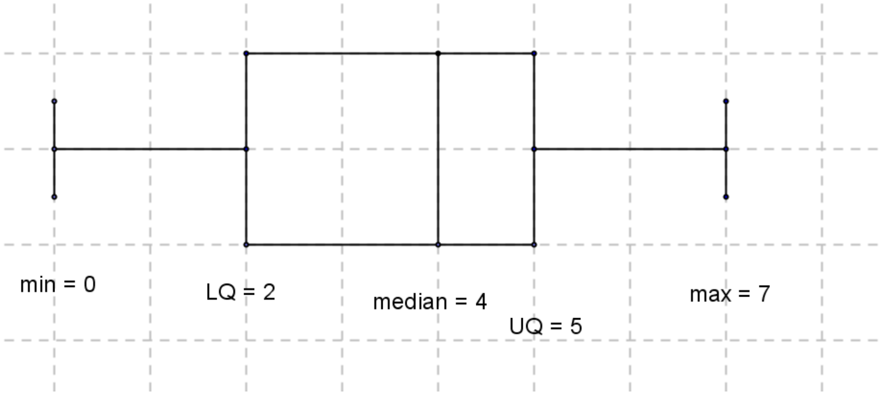
1. Read the first five lines of the text ending with "statistical applications".
2. Explain what the text deals with and comment on it.

Exercise

The question “ In a normal week, on how many days do you eat meat ? ” was asked to a class.
The results are :

1 ; 7 ; 0 ; 5 ; 4 ; 2 ; 1 ; 3 ; 4 ; 5 ; 6 ; 4 ; 1 ; 5 ; 3 ; 2 ; 3 ; 5 ; 4 ; 6 ; 1 ; 2 ; 6 ; 4 ; 4 ; 3 ; 4 ; 3 ; 4 ; 1.

1. How many students are in the class ?
2. The mean is 3.4 days. Explain how it was worked out.
3. Here is the the box and whisker plot that sum up the data.



Explain with words the meaning of the labels.

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SUJET 9 - CORRIGÉ

Box and whisker plot
Thème : Statistics

1. N = 30
2. Explications du calcul de la moyenne.
3. Si le candidat commente les indicateurs dans le cadre de la situation, lui demander comment on les calcule et réciproquement.

Tableau des données à titre indicatif

Numbers of days x_i	0	1	2	3	4	5	6	7	**total**
Number of students n_i	1	5	3	5	8	4	3	1	**30**
$n_i\, x_i$	0	5	6	15	32	20	18	7	**103**
Cumulative Number of students	1	6	9	14	22	26	29	30	

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SUJET 11

The Nine Chapters on the Mathematical Art

L’usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

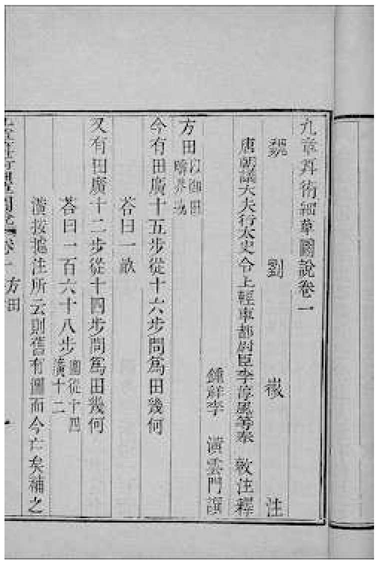
The Nine Chapters on the Mathematical Art is an ancient Chinese mathematics book, which was composed by several generations of scholars from the tenth to the second century BC. In the preface of this book, Liu Hui gave a detailed account of the history of the book, including the following sentences:

“When Zhou Gong[1] set up the rules for ceremonies, nine branches of mathematics were emerged, which eventually developed to the Nine Chapters of Mathematical art. Brutal emperor Qin Shi Huang burnt books, including the Nine Chapters. Later, in Han dynasty, Zhang Cang and Gen Shou Chang were famous for their mathematical skills. Zhang Cang and others re-arranged and edited the Nine Chapters of mathematical Art based on the damaged original text.”

The main theorem of Chapter 9 is the Gou Gu theorem which is known in the West as the Pythagorean Theorem.

Adapt from “DOCUMENTA MATHEMATICA (2012)

A page of *The Nine Chapters on the Mathematical Art*


九章算術細草圖說卷一
魏 劉徽 注
唐朝議大夫行太史令上輕車都尉臣李淳風等奉 敕注釋
鍾祥李 潢雲門譔
方田 以御田疇界域
今有田廣十五步從十六步問為田幾何
答曰一畝
又有田廣十二步從十四步問為田幾何
答曰一百六十八步 圖從十四廣十二
潢按據注所云則舊有圖而今亡矣補之

1. Read this text from the beginning down to “the following sentences”
2. Give a summary of the story of the book entitled « The nine Chapters of Mathematical Art”
3. What does the Pythagorean Theorem state?

[1] Zhou Gong is revered as one of the wise founding fathers of the Zhou (Chou) dynasty (ca. 1122–256 BCE); this era is considered like the golden age of China

Exercise

The chapter 7 of this book explains a way to solve the equation $ax+b=c$, where x has to be determined.

1. **1.** Show that $x=\dfrac{c-b}{a}$.

   a and *b* are not known, but what is given are two pairs $\{\alpha_1;\gamma_1\}$ and $\{\alpha_2;\gamma_2\}$ such that

   $$a\alpha_1+b=\gamma_1$$

   $$a\alpha_2+b=\gamma_2$$

   In this chapter it's said that subtracting the first equation from the second gives :

   $$a=\frac{\gamma_1-\gamma_2}{\alpha_1-\alpha_2}$$

2. **2.** Justify this result.
3. **3.** Multiplying the first equation by α_2 and the second equation by α_1 and subtracting these new equations gives : $b=\dfrac{\gamma_1\alpha_2-\gamma_2\alpha_1}{\alpha_2-\alpha_1}$. Justify this result.
4. **4.** Application : Use this method to solve the equation $ax+b=25$, knowing :

   $$3a+b=11$$

   $$9a+b=23$$

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SUJET 11 - CORRIGÉ

The Nine Chapters on the Mathematical Art

Summary

It tackles the book entitled: « The Nine Chapters of the Mathematical Art ».

This book dating from the 10th to the second century BC was the work of several Chinese scholars; it is said in the introduction that Zhou Gong was one of its major creators. Unfortunately this book, among others, was burnt by the Emperor Qin Shi Huang. Later on, it was re-written thanks to the help and skills of some mathematical scholars from Han dynasty. The most important theorem in Chapter 9 is the Gou Gu one also known as the Pythagorean Theorem in western countries.

The Pythagorean Theorem:

In a right triangle the square of the hypotenuse is equal to the sum of the squares of the two remaining sides. (Or the legs).

The solution of the system gives : $c = 25$, $\alpha_1 = 3$, $\gamma_1 = 11$, $\alpha_2 = 9$, $\gamma_2 = 23$.

And thus $x = 10$.

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SUJET 14

Thème : Probabilités

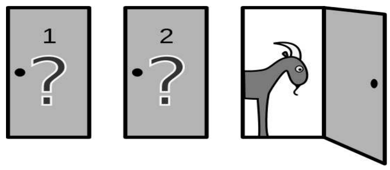
L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

I – The Monty Hall problem

The Monty Hall problem is a probability problem based on an American game show whose host was *Monty Hall*. It became famous when it was sent to *Marilyn vos Savant* in her "Ask Marilyn" column in *Parade magazine* in 1990. Here is the question :

Suppose you're on a game show, and the host lets you choose between three doors: Behind one door is a car; behind the two other doors there is a goat. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He then says to you, "Do you want to pick door #2?"

Vos Savant's response was that the contestant should switch to the other door but approximately 10,000 readers, including nearly 1,000 with PhDs, wrote to the magazine, most of them claiming vos Savant was wrong and that it did not change anything to switch doors.



1. Start by reading the last sentence of the text.
2. Explain what the text deals with and comment on it.

II –

We consider a bag containing four different letters : D, E, I and T.
We pick successively four letters at random in this bag, without replacement and range the letters from the left to the right. We obtain a word of four letters. This word has or has not a meaning in English.
We assume equiprobability.

1. Explain how to work out the number of words we can possibly obtain.
2. What is the probability to obtain a word starting by E ? Justify your answer.
3. What is the probability to obtain a word starting by DI ? Justify your answer.

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SUJET 14 - CORRIGÉ

Thème : Probabilités

I –
Reformuler les règles du jeu
Donner quelques éléments liés à la controverse.

II –

1. Toute démarche conduisant au calcul 4 X 3 X 2 1 = 24
2. Il y a 6 mots commençant par E. La probabilité d'obtenir un mot commençant par E est $\frac{6}{24} = \frac{1}{4} = 0,25$
3. Il y a 2 mots commençant par DI. La probabilité d'obtenir un mot commençant par E est $\frac{2}{24} = \frac{1}{12}$

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MATHÉMATIQUES – ANGLAIS

SUJET 16

Hippocrates' Squaring of a Lune
Thème : **Géométrie collège**

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Hippocrates of Chios was a mathematician and astronomer in Ancient Greece, he was born on the isle of Chios in 470 BC and died in 410 BC. He began as a merchant, then traveled to Athens, where he became a mathematician.
Hippocrates has often been described as a 'para-Pythagorean'. Hippocrates' major accomplishment was the writing of *Stoichia*, i.e. The Elements.
Over the course of the century following Hippocrates' death four other mathematicians wrote their own versions of *Elements.* Euclid's *Elements*, proved to be the culmination of Ancient Greece's geometric knowledge, and the text which remained the standard textbook of geometry for many centuries.
Of this founding work only a single fragment remains. The fragment deals with the Lune of Hippocrates, which was part of a research project on the calculation of the area of a circle, referred to as 'squaring the circle'.

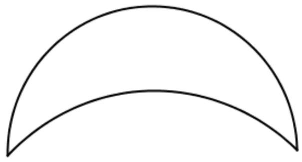
From http://www.egs.edu/library/hippocrates-of-chios/biography/

1. Start by reading the first paragraph of the text.
2. Sum up what you've read in the previous document.

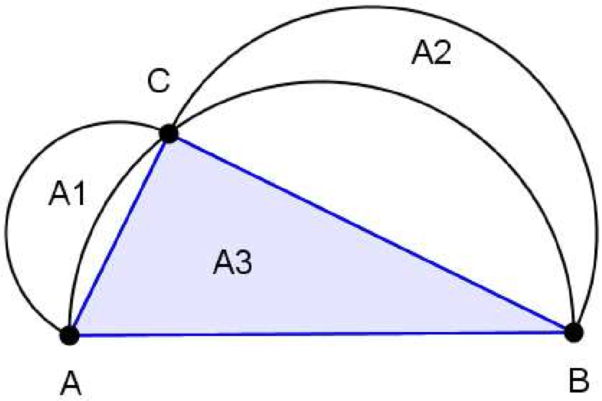
Exercise

Definition of a Lune :

A Lune is a figure in a plane that is bounded by two circular arcs as shown below.



1. Explain, step by step, how you would draw a lune.
2. The aim of this exercise is to prove that, in the figure below, the sum of the area of the Lune ended up at A and C and the area of the Lune ended up at B and C is equal to the area of triangle ABC.



Which means that A1 + A2 = A3

Where A1 is the area of the Lune ended up at A and C, A2 is the area of the Lune ended up at B and C and A3 is the area of triangle ABC.

Explain how it is possible to find the sum of the areas of the two Lunes A1+A2, using the area of triangle ABC and the area of the semi-circle of diameter AB.

3. Prove then that the sum of the area of the Lune ended up at A and C and the area of the Lune ended up at B and C is equal to the area of triangle ABC.

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MATHÉMATIQUES – ANGLAIS

SUJET 16 - CORRIGÉ

Hippocrates' Squaring of a Lune
Thème : Géométrie collège

Éléments à prendre en compte pour évaluer la capacité d'analyse :

Analyse du texte :

- Text about the Hippocrates of Chios
- 1st paragraph: Dates of birth and death, country, where he was born,
- 2nd and 3rd paragraph: He wrote a book called “the Element”, rewritten later by Euclid, all the knowledge of mathematics at this time was in this book.
- 4th paragraph: In Euclid’s Elements we can find one work of Hippocrates: Squaring the Lunes of Hippocrates which is demonstrated in the exercise below.
- Extensions :
  - Euclidean geometry
  - Euclid

Éléments à prendre en compte pour évaluer la capacité d'argumentation :

- Question 1 de l’exercice : description de la construction de la lunule.

Exercise

1. The student should say that he draws two circular arcs (ex: a semi-circle, quarter of circle…) and he should give their center and radius. Then he should show by what the Lune is bounded.
2. A1+A2 = area of semi-circle of diameter AC + of semi-circle of diameter BC - of semi-circle of diameter AB + area of triangle ABC.
3. A1+ A2 = $\frac{1}{2}\times\pi\times\left(\frac{AC}{2}\right)^2+\frac{1}{2}\times\pi\times\left(\frac{BC}{2}\right)^2-\frac{1}{2}\times\pi\times\left(\frac{AB}{2}\right)^2$ + area of triangle ABC

   A1 + A2 = $\frac{1}{8}\times\pi\times\left(AC^2+BC^2-AB^2\right)$ + area of triangle ABC.

   But ABC is a right angled triangle in C so, using Pythagoras’ theorem, $AB^2=AC^2+BC^2$

   Thus, $AC^2 + BC^2 - AB^2 = 0$

   And A1 + A2 = area of triangle ABC.

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MATHÉMATIQUES – ANGLAIS

SUJET 19

Falling bodies : from Galileo to Newton
Topic : Basic functions

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Galileo (1564-1642) used traditional mathematical methods in an innovative way for his theoretical and experimental work on the motions of bodies and was then a precursor of the classical mechanics developed by Sir Isaac Newton (1643-1727). On the contrary, the latter created new mathematical methods, such as infinitesimal calculus, to develop his theories. Both of them worked on celestial mechanics, optics, falling bodies and so many other topics.

Alexander Pope wrote the famous epitaph : Nature and nature's laws lay hid in night ; God said « Let Newton be » and all was light .
However, Newton himself modestly wrote about his achievements « If I have seen further, it is by standing on the shoulders of giants »

1. Read the first paragraph aloud.

2. Point out the main ideas of this text and develop them

Exercise

Galileo described an experiment to prove that the mass of a body didn't influence its falling speed. With the help of Newton's gravitation theory, when studying a ball falling from one of the highest windows of the Pisa tower, the distance *d* travelled by the ball in a time *t* is given by the function : $d(t)=\frac{1}{2}gt^2$, where g ≈ 9,81 m.s^{-2} is the standard gravity, *d* is in meters, and *t* in seconds.

Answer the following questions.

1. What type of function is *d* ? How do you call the curve of such a function ?
2. What is the image of 2 under the function *d* ?
3. What is 2 for the previous result, then ?
4. Complete the following table of values:

t in seconds	0	0,5	1	2	3	4	5
d in meters							

5. The window from which the ball fell was around 51 m high above the ground. How long does it take to hit the ground for the first time ?
6. Explain how you could measure the height of a building using such a function.

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MATHÉMATIQUES – ANGLAIS

SUJET 19 - CORRIGÉ

Falling bodies : from Galileo to Newton
Topic : Basic functions

2. The student should point out one or two things about Galileo. His improvement of the refracting telescope, with which he was able to make many celestial observations leading him to agree with Copernicus theory : heliocentrism. This caused him lots of troubles with the catholic church defending the geocentric theory : he was trailed and sentenced to formal imprisonment by the Inquisition. A second idea, hinted in the text and the exercise, is about his experiments about falling bodies. He described an experiment to prove, contrary to Aristotle, that a heavy body or a light one fall at the same speed : just let two bodies of different mass fall from the Pisa Leaning Tower and record that they reach the ground at the same time.

About Newton, the student should point out that he pushed further lots of scientific ideas like the motion of (celestial) bodies with the theory of universal gravitation, or just the little story about the apple and the notion of gravity. The student could point out his work about infinitesimal calculus, basis of today's derivative.
With Alexander Pope's epitaph, it should also be pointed out that this theory of universal gravitation, and others like the theory of decomposition of light, were major discoveries to understand a lot better nature's laws.

The student could conclude with a comment on the last sentence and how scientific discoveries may occur.

1. *d* is a quadratic function. Its curve is a parabola. In fact, just half of it, as *t* is positive only.
2. The image of 2 under *d* is d(2)=1/2*g*2²=2g ≈ 19,62.
3. 2 is then the counter-image (or pre-image) of 2*g*.
4.

t in s	0	0,5	1	2	3	4	5
d in m	0	$\frac{g}{8} \simeq 1.22625$	$\frac{g}{2} \simeq 4.905$	$2g \simeq 19.62$	$\frac{9}{2}g \simeq 44.145$	$8g \simeq 78.48$	$\frac{25}{2}g \simeq 122.625$

Beside is the curve of *d* in function of *t*.

5. Graphically, we can say that it takes around 3.2 seconds for the ball to hit the ground. But we can also calculate it solving the equation : $\frac{1}{2}gt^2 = 51$. The solution is t=racine(102/g) ≈ 3,22 s. This last result is congruent with the first approximation.
6. To measure the height of a building, you just need to time the fall of a body, say a stone. You then need to square this time, multiply it by 9.81 and divide it by 2 to have the height of the building in meters.

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MATHÉMATIQUES – ANGLAIS

SUJET 1

Sports and mathematics
Thème : Probabilités

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Aristotle (384 BC – 322 BC) said : "*The probable is what usually happens*". You can't predict the future, but you can use mathematical probability to determine how likely it is that something will – or won't – happen.
People often use probability to make decisions in a wide range(1) of fields, because you can use it whenever you have to calculate risks.
For example, probability is used to predict if it's more likely to rain tomorrow or not as well as to prognosticate the result of a sporting event if you want to bet on it. You can even use it to decide whether you should undergo surgery(2) or not.
The range(1) of possibilities is huge.

Adapted from the website : http://www.ehow.com/list_7719506_real-life-probability-examples.html

1. Start the interview by reading the first three lines of the text ending with "happen".
2. Explain what the text deals with and comment on it.

Exercise

James practises archery(3). His performances are influenced by the weather.
If there is no wind, he hits the centre of the target one time out of three. However, if it's a windy day, he hits the centre of the target only one time out of eight.
In 2012 (366 days), there were 54 windy days when James practised his sporting activity.
All the following questions are referring to the year 2012.

1. Draw a tree diagram representing the situation.
2. If it's a windy day, what is the probability that he doesn't hit the centre of the target?
3. What is the probability that he hits the centre of the target?
4. If we know that James hits the centre of the target, what is the probability that it's a windy day?

(1) range : étendue
(2) undergo surgery: se faire opérer
(3) archery : tir à l'arc

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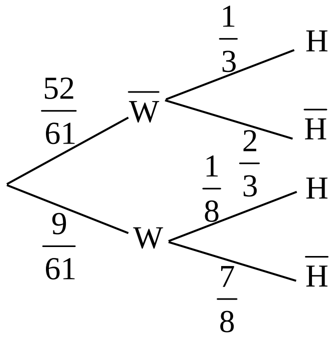
MATHÉMATIQUES - ANGLAIS

SUJET 1 – CORRIGÉ

Sports and mathematics – sujet 1
Thème : Probabilités

Exercise

1. Let's consider the following events :
W : "It's a windy day"
H : "James hits the centre of the target"



$P(W)=\frac{54}{366}=\frac{9}{61}$.

2. $P_W(not\ H)=1-\frac{1}{8}=\frac{7}{8}$. If it's a wi that he doesn't hit the centre of the target is $\frac{7}{8}$ $(=87,5\%$

3. $P(H)=P(not\ W\cap H)+P(W\cap H)=P(not\ W)\times P_{not\ W}(H)+P(W)\times P_W(H)$

$P(H)=\frac{52}{61}\times\frac{1}{3}+\frac{9}{61}\times\frac{1}{8}=\frac{52}{183}+\frac{9}{428}=\frac{443}{1,464}$.

The probability that he hits the centre of the target is $\frac{443}{1,464}$ $(\approx 30,3\%)$.

4. We are looking for $P_H(W)$.

$P_H(W)=\frac{P(W\cap H)}{P(H)}=\frac{9}{488}\div\frac{443}{1,464}=\frac{9}{488}\times\frac{1,464}{443}=\frac{27}{443}$.

If we know that James hits the centre of the target, the probability that it's a windy day is $\frac{27}{443}$ $(\approx 6,1\%)$ $\frac{27}{443}$.

<u>Éléments à prendre en compte pour évaluer la capacité d'analyse et d'argumentation :</u>

- Interpréter les phrases du texte en termes de probabilités.
- Utiliser un vocabulaire adapté au niveau des probabilités et pour la lecture de dates (1ère phrase du texte).
- Utiliser les probabilités conditionnelles.

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MATHÉMATIQUES – ANLGAIS

SUJET 2

Theater Seating
Thème : Sequences and series

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

There is no specific history as to when sequences were started although there was a young math student who created a formula to help solve the sum of arithmetic sequences. His name was Carl Gauss, he was born in 1777 in the German Empire and at just ten years old he created this formula. As Gauss grew older he became a very well-known mathematician contributing to geometry, number theories, and many more.

Sequences and series are applied with many different things. You can use them to arrange seating capacity in an auditorium or theater. They could also be used to help build many things such as a patio, a fence, and a lot of other different structures. Businessmen also often use sequences and series to learn about their profits and what they have made over the last years and the differences and then they try to use that information to predict future sales.

Extract adapted from Rabun Gap Algebra III

1. Read the first five lines of the text ending with theater.
2. Comment on the text.

Exercise

A theater has 40 seats in the first row, 42 seats in the second row, 44 in the third row, and so on.

1. What type of sequence can you use to modelize this situation ? Justify.

2. Find a symbolic representation for a sequence a_n that gives the number of seats in row n.

3. How many seats are there in row 20?

4. Find the total number of seats from row 1 to row 20 in this theater.

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MATHÉMATIQUES – ANGLAIS

SUJET 2 - CORRIGÉ

Theater Seating
Thème : Sequences and series

Éléments à prendre en compte pour évaluer la capacité d'analyse et d'argumentation :

- L'élève pourra faire un résumé des informations du texte et insister sur les divers domaines d'application des suites numériques.

- Dans l'exercice, l'élève sera amené à argumenter son choix en comparant suite arithmétique et suite géométrique.
- L'utilisation d'un vocabulaire spécifique aux suites est souhaitable.

Corrigé de l'exercice

1. It can't be modeled by a geometric sequence because the ratio of two consecutive terms is not always the same. For example 42/40 = 1.05 whereas 44/42 =1.04742. However, when we compute the difference between the numbers of seats in two consecutive rows, it always gives 2. This can thus be modeled by an arithmetic sequence which common difference is 2.

2. As in first row there are 40 seats, let's say that $a_1 = 40$. Then the recurrent rule for this sequence is $a_{n+1} = a_n + 2$. So the nth term is given by $a_n = a_1 + (n - 1)\, d$ where $d = 2$. Finally $a_n = 40 + 2(n - 1)$ so $a_n = 40 + 2n - 2 = 38 + 2n$.

3. Term a_{20} has to be computed to find the answer.
$a_{20} = 38 + 2 \times 20 = 78$.
There are 78 seats in row 20.

4. To find the total number of seats in this theater, we have to compute the sum of the twenty first terms of this arithmetic sequence. We know that the formula is :
$a_1 + a_2 + \ldots + a_n = \left(\frac{a_1 + a_n}{2}\right)\times n$. Here we have to compute $\left(\frac{40+78}{20}\right)\times 20 = 1180$.
In this theater, there are 1180 seats.

5. (extension possible) The theater is extended up to 1500 seats. How many rows should be added?