BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 1

Theme: Statistics

L’usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Sir Francis Galton (1822-1911) was an English Victorian progressive, polymath, anthropologist, eugenicist, tropical explorer, geographer, inventor, meteorologist, proto-geneticist, and statistician. He was knighted in 1909.
He was the first to apply statistical methods to the study of human differences and inheritance of intelligence, and introduced the use of questionnaires and surveys for collecting data on human communities.
Galton was a keen observer. In 1906, visiting a livestock fair, he stumbled upon an intriguing contest. An ox was on display, and the villagers were invited to guess the animal's weight after it was slaughtered and dressed. Nearly 800 participated, but not one person hit the exact mark: 1,198 pounds. Galton stated that "the middlemost estimate expresses the *vox populi*, every other estimate being condemned as too low or too high by a majority of the voters", and calculated this value (in modern terminology, the median) as 1,207 pounds. To his surprise, this was within 0.8% of the weight measured by the judges. Soon afterwards, he acknowledged that the mean of the guesses, at 1,197 pounds, was even more accurate.

From http://en.wikipedia.org/wiki/Francis_Galton

1. Start the interview by reading the 5 last four lines from “Galton stated that…” down to “more accurate”

2. Explain what the text deals with and comment on it

Exercise

1) In January of 2014, your family moved to a tropical climate. For the year that followed, you recorded the number of rainy days that occurred each month. Your data contained 14, 14, 10, 12, 11, 13, 11, 11, 14, 10, 13, 8.

a. Find the mean, mode and median for your data set of rainy days.

b. Explain the easiest way to calculate the mean, mode and median for the 2015 data in the two following cases:

- If the number of rainy days doubles each month in the year 2015.
- If, instead, there are three more rainy days per month in the year 2015.

2) A storeowner kept a tally of the sizes of suits purchased in her store. Which measure (mean, median or mode) should the storeowner use to describe the average size suit sold?

3) A tally was made of the number of times each color of crayon was used by a kindergarten class. Which measure (mean, median or mode) of central tendency should the teacher use to determine which color is the favorite color of her class?

4) The science test grades are posted. The class did very well. All students taking the test scored over 75. Unfortunately, 4 students were absent for the test and the computer listed their scores as 0 until the test is taken. Assuming that no score repeated more times than the 0's, what measure (mean, median or mode) of central tendency would most likely give the the best representation of this data?

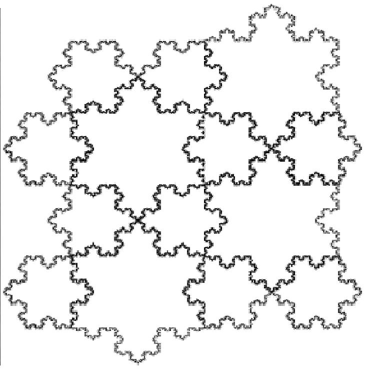
Sources : http://www.regentsprep.org/regents/math/algebra/AD2/Pmeasure.htm

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

<u>SUJET 2</u>

Theme: Sequence

L’usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Niels Fabian Helge von Koch (25 January 1870 – 11 March 1924) was a Swedish mathematician who gave his name to the famous fractal known as the Koch snowflake, one of the earliest fractal curves to be described.

He was enrolled at the newly created Stockholm University College in 1887 and at Uppsala University in 1888. He received his Ph.D. in Uppsala in 1892. He was appointed professor of mathematics at the Royal Institute of Technology in Stockholm in 1905.

Von Koch wrote several papers on number theory. In 1901, Von Koch published *On the Distribution of Prime Numbers,* which concentrated on the prime number theorem. In 1906, he released his work on curves and snowflakes

The Koch Curve

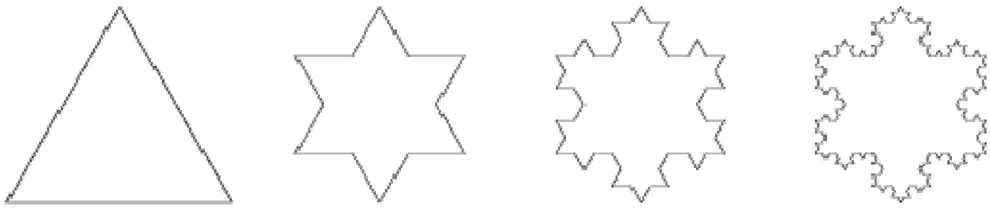
In order to create the Koch Snowflake, Von Koch began with the development of the Koch Curve. The Koch Curve starts with a straight line that is divided up into three equal parts. Using the middle segment as a base, an equilateral triangle is created. Finally, the base of the triangle is removed, leaving us with the first iteration of the Koch Curve.

From Wikipedia, the free encyclopedia.

1. Read the first five lines of the text ending with "Stockholm in 1905"
   Explain what the text deals with and comment it.

2. **Exercise**
   The snowflake curve is generated from the side of an equilateral triangle whose side is 1 (**Step 1**), splitting in 3 parts each of length $\frac{1}{3}$ and adding a triangle in the middle position. (**Step 2**).

   The steps in creating the Koch Curve are then repeatedly applied to each side of the equilateral triangle, creating a "snowflake" shape.



Step 1 **Step 2** **Step 3 ……………… Step n**

Let C_n be the **number of sides** at step n. ($n > 0$)

Let P_n be the **length of the <u>perimeter</u>** at step n.

1) Give the value of C_1 and P_1.

2) a) Give the value of C_2 , C_3 and C_4.
   b) Express (C_n) as a geometric sequence. Explain.

3) a) Prove that $P_2 = 4n$.

4) Explain why : $P_n = 3\left(\frac{4}{3}\right)^{n-1}$

5) What happens as we let n tend to infinity? What do you think about this?

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

<u>SUJET 3</u>

Theme: Arithmetics

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

PERFECT NUMBERS

Those ten simple symbols, digits, or numbers that we all learn early in life influence our lives in far more ways than we could ever imagine. Have you ever wondered what our lives would be like without these 10 elegant digits and the infinite array of other numbers that they can create? Birthdays, ages, height, weight, dimensions, addresses, telephone numbers, license plate numbers, credit card numbers, PIN numbers, bank account numbers, radio/TV station numbers, time, dates, years, sports scores, prices, sequences/series of numbers, magic squares, polygonal numbers, Fibonacci numbers, perfect numbers, and the list goes on ad infinitum. Engineers, manufacturers, cashiers, bankers, carpenters, mathematicians, scientists, and so on, could not survive without them.

From Mathgoodies.com

1. Read the first seven lines of the text ending with ad infinitum.
2. Comment on the text.

<u>Exercise</u>

A perfect number is an integer bigger than one which meets the condition that the sum of its positive factors except the number itself is equal to the number.

1. 6 = 3 + 2 + 1. What does this statement prove and why?
2. Is 12 a perfect number? Explain.
3. Prove that 28 is a perfect number.
4. Look at the pattern which those numbers follow :
   $6 = 2 \times (2^2 - 1)$ *where* $(2^2 - 1)$ *is prime.*
   $28 = 2^2 \times (2^3 - 1)$*where* $(2^3 - 1)$ *is prime.*

   Describe this pattern and try to use it to discover the two following perfect numbers.

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 4

Theme: Geometry

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

The origins of Trigonometry

In the first centuries of the Common Era, trigonometry began with the works of Greek astronomers, such as Hipparcus, Ptolemy and Menelaus, who were interested in the way to calculate the length of a chord, given the length of the circular arc joining its endpoints. It was Indian mathematicians and astronomers, especially Surya Siddhanta (c. 300 – 400 AD), who worked out what we now call the functions sine and cosine. From India the sine function was introduced to the Arab world in the 8th century, where the Sanskrit term *jya* was translated into *jiba* or *jyb*. Several centuries later, it was transmitted to Europe through contacts among merchants. The Europeans mistook the Arabic word *jiba* for *jaib* (the opening of a woman's garment at the neck), which was translated into the Latin *sinus*, which can mean "fold" (in a garment), or "curve", hence the word "sine"!

Adapted from: The birth of Trigonometry by Dr Laura A.Smoller,
University of Arkansas at Little Rock, Department of History.

1. Read the first paragraph out loud.

2. Give a brief account of the text.

3. The Johnstown Inclined Plane in Pennsylvania is one of the longest and is considered the "world's steepest vehicular inclined plane". Even though the railway itself is a little longer, the cable cars travel a distance of 867.1 feet. The angle of the runaway is 35°25' and the elevation of the top landing is 1693.5 feet.

   a) Verify that $35°25' \approx 35.4°$.

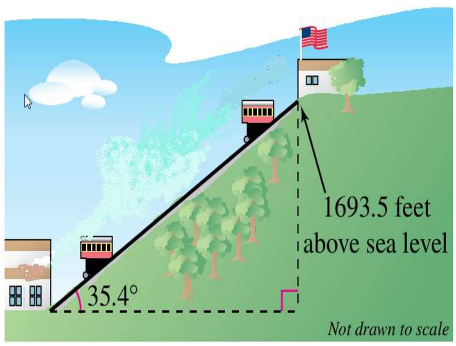
   b) Work out the vertical rise of the railway cars.

   c) What is the elevation of the bottom landing?

   d) The cars move up the mountain at a rate of 300 feet per minute. How long is the trip?

   e) The road sign below is used in US, it shows that the road has a 6% grade, which means that over a 100 feet horizontal distance, the vertical rise is 6 feet.

If the Johnstown Inclined Plane were a road, what percentage would you read on the road sign?



BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 5

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

"Teens Today Don't Read Books Anymore"

Are teens really not reading as much as they did in the past? Many studies are only taking into account traditional books and print-based reading.

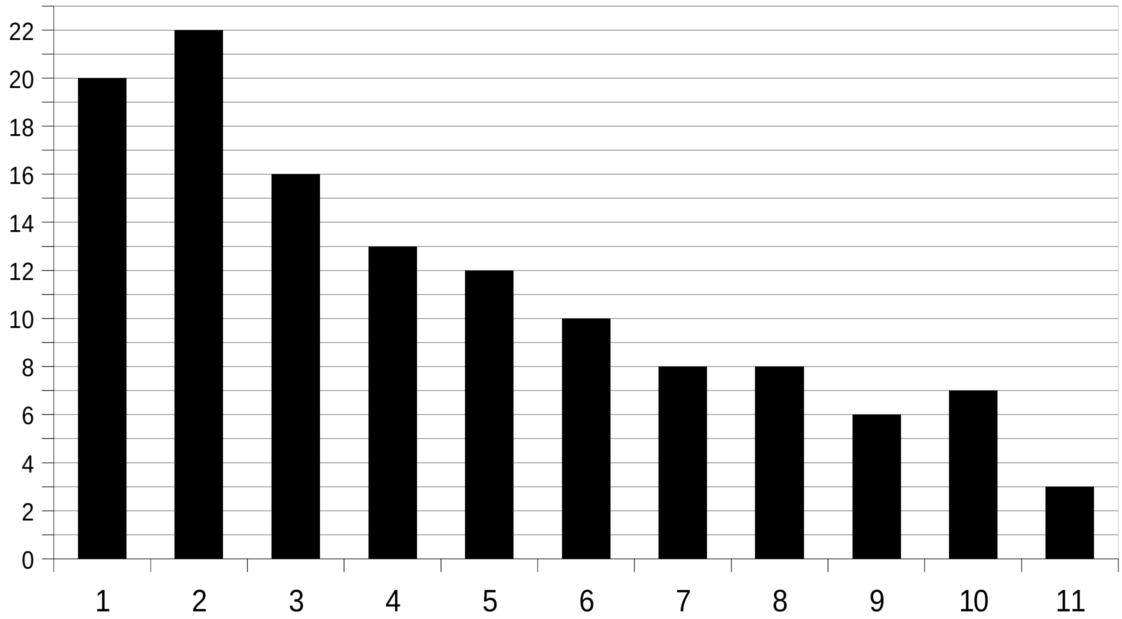
A USA Today story from 2007 was headlined about young adults: "One in four read no books last year". Also in 2007, the National Endowment for the Arts (NEA) conducted a follow-up, compendium, and analysis of reading focusing on children called "To Read or Not to Read". Results indicated that children and young adults were reading significantly less than in the past: "Less than one third of 13-year-olds are daily readers, a 14 percent decline from 20 years earlier. Among the 17-year-olds, the percentage of non-readers doubled over a 20-year period, from nine percent in 1984 to nineteen percent in 2004. On average, Americans aged 15 to 24 spend almost two hours a day watching TV, and only seven minutes of their daily leisure time reading."

If nearly all Internet surfing and social networking is text-based, how can this be true? Would it have been similar if the teens from previous decades had had access to the Internet?

Abridged from a post by Jessica E. Moyer (assistant professor in the School of Information Studies at the University of Wisconsin Milwaukee) on November 2. 2010 in Young Adult Library Services Association (YALSA) website (www.yalsa.ala.org).

1. Read the article from "Less than one ... to ... leisure time reading."
2. Explain what the text deals with and comment on it.

In 2004,125 students have been questioned about the number of books they read in one year. The following chart summarizes the results:



1. How many students read precisely three books?

2. Which percentage of students read three books or less?

3. What is the mode?

4. Using your personal calculator, compute the mean of this set. Knowing that it decreased by 35 % in ten years, what was the mean in 2014?

5. Explain why the median number of books read by students is 4.
What does it mean?

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 7

Theme: Maths and music: sequences, ratios, equalities

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Part I

Leonardo Bonacci (c. 1170 – c. 1250) known as Fibonacci, and also Leonardo of Pisa, was an Italian mathematician, considered to be "the most talented Western mathematician of the Middle Ages." He introduced to Europe, the Fibonacci numbers or Fibonacci sequence which are the numbers in the following integer sequence:

1 ,1 ,2 ,3 ,5 ,8 ,13 ,21 ,34 ,55 ,89 ,144 ,…

They are intimately connected with the golden ratio (two quantities *a* and *b* are said to be in the golden ratio φ if : $\frac{a+b}{a} = \frac{a}{b} = \varphi$, φ is named the golden number*),*for example, the closest rational approximations to this ratio are : 2/1 ; 3/2 ; 5/3 ; 8/5;…..

A study has revealed examples of the golden proportion in various musical periods. A well-known example is the "Hallelujah" chorus in Handel's Messiah. Whereas the whole consists of 94 measures, one of the most important events (entrance of solo trumpets) happens in measures 57 to 58, after about 8/13 of the whole piece. In addition to that, one can find a similar structure in both of the division of the whole piece. After 8/13, of the first 57 measures, that is in measure 34, the entrance of the theme marks another essential point, and so on…

Extracts from Wikipedia and from the essay "how do mathematics and music relate to each other" by Michael Beer

1. Read the first paragraph of the text.
2. What does the text deal with?

Part II:

1) Could you find the pattern of Fibonacci numbers?

2) Using the golden ratio of the text, through simplifying the fraction and substituting in b/a = 1/φ, could you prove that: $1 + \frac{1}{\varphi} = \varphi$?

3) Multiplying by φ, prove that : $\varphi^2 - \varphi - 1 = 0$.

(With that equation and the quadratic formula, we can prove that: $= \frac{1+\sqrt{5}}{2}$)

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 8

Theme: Conditional probabilities

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Have you ever had that anxiety dream where you suddenly realize you have to take the final exam in some course you've never attended? For professors, it works the other way around, you dream you're giving a lecture for a class you know nothing about. That's what it's like for me whenever I teach probability theory. It was never part of my own education, so having to lecture about it now is scary and fun, in an amusement park, thrill-house sort of way.

Perhaps the most pulse-quickening topic of all is "conditional probability", the probability that some event *A* happens, given (or "conditional" upon) the occurrence of some other event *B*. It's a slippery concept, easily confused with the probability of *B* given *A*. They're not the same, but you have to concentrate to see why.

Adapted from Chances Are, a post by Steven Strogatz, New York Times' Opinion Pages

In the USA, students in a high school can choose to study Spanish or French as a foreign language. In a given academic year, 90% of the pupils choose Spanish and the rest choose French. 30% of the students who learn Spanish are boys and 40% who study French are also boys. We choose a random student in this high school.

Let F be the event: the student studies French
Let S be the event: the student studies Spanish
Let B be the event: the student is a boy.

1) Represent the situation with a probability tree.

2) What is the probability that the student is a boy knowing that the student learns Spanish?

3) What is the probability that the student is a boy and learns Spanish?

4) What is the probability that the student is a boy?

5) What is the probability that the student learns Spanish knowing that he is a boy?

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 9

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

QR code (abbreviated from Quick Response Code) is the trademark for a type of matrix barcode (or two-dimensional barcode) first designed for the automotive industry in Japan. A barcode is a machine-readable optical label that contains information about the item to which it is attached. A QR code uses four standardized encoding modes (numeric, alphanumeric, byte / binary, and kanji) to efficiently store data; extensions may also be used.

A QR code consists of black modules (square dots) arranged in a square grid on a white background, which can be read by an imaging device (such as a camera) and processed using Reed–Solomon error correction until the image can be appropriately interpreted. The required data are then extracted from patterns present in both horizontal and vertical components of the image.

Here is an example of QR code

1) How many possible QR codes are there?
2) Give the probability of a totally white QR code?
3) Give the probability of an half blacked QR code.

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 10

Thème: Les suites

L’usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Introduction

Core samples, tide gauge readings, and, most recently, satellite measurements tell us that over the past century, the Global Mean Sea Level (GMSL) has risen by 4 to 8 inches (10 to 20 centimeters). However, the annual rate of rise over the past 20 years has been 0.13 inches (3.2 millimeters) a year, roughly twice the average speed of the preceding 80 years.

Over the past century, the burning of fossil fuels and other human and natural activities has released enormous amounts of heat-trapping gases into the atmosphere. These emissions have caused the Earth's surface temperature to rise, and the oceans absorb about 80 percent of this additional heat.

The rise in sea levels is linked to three primary factors, all induced by this ongoing global climate change:

- **Thermal expansion:** When water heats up, it expands. About half of the past century's rise in sea level is attributable to warmer oceans simply occupying more space.
- **Melting of glaciers and polar ice caps:** Large ice formations, like glaciers and the polar ice caps, naturally melt back a bit each summer. (…)
- **Ice loss from Greenland and West Antarctica:** As with glaciers and the ice caps, increased heat is causing the massive ice sheets that cover Greenland and Antarctica to melt at an accelerated pace.

When sea levels rise rapidly, as they have been doing, even a small increase can have devastating effects on coastal habitats. (…) In addition, hundreds of millions of people live in areas that will become increasingly vulnerable to flooding. Higher sea levels would force them to abandon their homes and relocate. Low-lying islands could be submerged completely.

« Sea level rise », National Geographic

Exercise

If we take as a reference the level of the sea 4000 years ago, its elevation is around 1 meter since that period; that phenomenon started very slowly but during the 20th century, it was accelerated. Between 1995 and 2010, scientists estimated that the sea rose by around 50 mm.

1) A first hypothesis into guessing what the level of the sea would be in 2100 is to estimate that this elevation will continue in a similar way by adding 50mm for every 15 years.

So if we call u_0 the level of the sea in 2010 in meter, we can say: $u_0 = 1$. u_1 represents the level in 2025, u_2 the level in 2040 and so on…

a) Calculate u_1 and u_2.
b) What sort of sequence is (u_n)? Express u_n in terms of n.
c) Calculate u_6. What would be the level of the sea in 2100 according to this hypothesis?

2) A second hypothesis consists in saying that the level of the sea will rise by 12,25% every 15 years starting from 1 meter in 2010. So we admit: $v_0 = 1$ and now we call v_n the level of the sea after n periods of 15 years.

a) What sort of sequence is (v_n)? Show that $v_n = 1.225^n$.
b) Calculate v_6. Compare $v_6 - v_0$ and $u_6 - u_0$. What do you think of these results knowing that, in the world today, 150 million people are living between sea level and 0.5m above sea level.

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 11

Thème: Arithmétique

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Properties of the number 2015

Let's learn a little bit more about the arithmetical properties of the number 2015!

Well, to begin with, let's notice that since its ones digit is a five, 2015 can be divided by 5: therefore, it is not a prime number but a composite number, and we can even write: 2015 = 5 * 13 * 31. But as a composite integer, it has interesting properties:
First of all, it is a **sphenic number**.

A sphenic number is a positive integer that is the product of exactly three distinct prime numbers. For instance, 1001 = 7 * 11 * 13 is a sphenic number, but 60 = 2^2 * 3 * 5 is not a sphenic number, because those must be square-free. Curiously enough, 2013 and 2014 are also sphenic numbers! We can also notice that a sphenic number always has exactly eight divisors.

But 2015 is also a **Lucas-Carmichael number**, which is much rarer. In fact, it is only the third smallest such number after 399 and 935.
A Lucas-Carmichael number is a positive composite integer n with distinct factors and such that if p is a prime factor of n, then p+1 is a factor of n+1.

Adapted from various Wikipedia pages

1. Read the paragraph about sphenic numbers, from « *A sphenic number…* » to « … *eight divisors* »

2. Explain what the text deals with.

3. What is a prime number?

4. *"Since its ones digit is a five, it can be divided by 5"*
   a) Explain this sentence
   b) Is the conversion of this proposal also true?

5. According to the definition, what would be the smallest sphenic number?

6. *"A sphenic number always has exactly eight divisors."*
   a) Check this statement for 2015.
   b) Prove this statement.

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 12

Thème: Fonctions

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Logarithms

The sixteenth and seventeenth centuries saw important advances in the tools and language of mathematics. Napiers's invention of logarithms radically reduced the labour involved in complex calculations.

Logarithms are important to the History of mathematics and related disciplines because they revolutionized the laborious and difficult process of calculation, offering the power of the modern pocket calculator before such a thing existed. A contemporary university website describes the logarithm as "perhaps the single, most useful arithmetic concept in the sciences", while the French astronomer and mathematician Pierre-Simon Laplace (1749 – 1827) remarked that the invention of logarithms had doubled the lifetimes of astronomers by halving their labours.

The invention of logarithms sprang from a desire to make multiplication and division as simple as addition and subtraction. This is possible where an arithmetic series corresponds to a geometric series.

From A curious history of mathematics by Joel Levy

1. Read the second paragraph of the text.
2. Explain what the text deals with and comment on it.

Exercise

Considering the following table, answer the questions:

x	0.1	0.5	1	1.5	2	3	4	5	6
$f(x)$					0.69 315	1.09 861	1.38 629	1.60 944	1.79 176

x	7	8	9	10	11	………..	21	22
$f(x)$	1.94 591	2.07 944	2.19 722	2.30 259	2.39 790			

1. To the product of two numbers of the first row corresponds the sum of the corresponding numbers in the second row. Check this property with some examples.

2. Which number must be written below 1? Explain.

3. Which number must be written below 21? Below 22?

4. Which operation corresponds to the division of two numbers of the first row? Check your answer using an example.

5. Deduce the numbers we have to write below 0.1; 0.5 and 1.5.

6. Explain why 0 can't have any image under *f*.

7. What is the image of 3^5 under *f*?

BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 13

Theme: Geometry and algebra

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

Aspect Ratios of Monitors Screen

The screens used in the computer industry have evolved towards particular shapes and sizes over the past twenty years. The `size', as we had from the start with TV screens, is labeled by the length of the diagonal between opposite top and bottom corners of the monitor screen. The shape is defined by an `aspect ratio' which gives the ratio of the width to the height of the screen.

Before 2003, most computer monitors had an aspect ratio of 4 to 3. From 2003 until 2006 the industry moved towards an office standard of 16 to 10, that was less square and more `landscape' in dimension. This ratio is almost equal to the famous `golden ratio' of 1.618, which is presumably no accident. It has often been claimed by architects and artists to be aesthetically pleasing to the eye.

Adapted from "100 essential things you didn't know you didn't know about Math and the Arts" by John D. Barrow. (2014)

1. Read the first paragraph of the text.
2. Comment on the text.

Exercise

1) A screen has an aspect ratio of 5 to 4. Its width is 15 inches.
   a) What is the height of the screen?
   b) What is the size of the screen?
2)
   a) A screen has a size of 20 inches and an aspect ratio of 4 to 3. What are its width and its height?
   b) Is it possible to work out the width and the height of a screen whose size is known but not its aspect ratio?
3) In a store, you notice 2 screens S_1 and S_2 totally identical: same size (22 inches), same price, same technical characteristics, except for their aspect ratio : the screen S_1 has an aspect ratio of 4 to 3 while the other one has an aspect ratio of 5 to 4. You need to get a screen with the largest area. Which one will you choose? Explain your reasoning.